coordinate system the inverse problem becomes a direct problem. Our results for thin bodies at low Mach numbers show there is a pressure minimum on the bodies, which can imply flow separation in some cases.

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Forced Convection over Rotating Bodies with Blowing and Suction

Fue-Sang Lien* and Cha'o-Kuang Chen†
National Cheng Kung University
Tainan, Taiwan, China
and
John W. Cleaver‡
University of Liverpool
Liverpool, England, United Kingdom

Introduction

SPINNING an axisymmetric body in a forced flow field in order to develop rotating systems for enhancing the heat-transfer rate is important in the analysis of problems involving projectile motion and rotary machine design. Several studies of this problem[1,2] have yielded very effective solutions for moderate or high Prandtl numbers and for small values of the rotation parameter. In order to avoid the difficulties encountered in previous methods, Lee et al.[3] have analyzed the momentum and heat-transfer rates through laminar boundary layers over rotating isothermal bodies by employing Merk's series expansion technique.

Received Sept. 13, 1984; revision submitted Sept. 18, 1985.

*Graduate Student, Department of Mechanical Engineering.
†Professor, Department of Mechanical Engineering.
‡Senior Lecturer, Department of Mechanical Engineering.

The present study is an extension of Ref. 3 and deals with an isothermal (ISO) or constant-heat-flux (CHF) rotating body with blowing and suction. An efficient finite difference method is employed. These results are compared with those of Lee et al.[3] and Hoskin.[2]

Analysis

This paper is an extension of Ref. 3 to include injection and suction at the surface. The basic governing equations are identical to those of Ref. 3 with the following boundary conditions:

$$u=0, \quad v=v_w, \quad \omega=r\Omega, \quad T=T_w$$

$$\text{or} \quad k\frac{\partial T}{\partial y}=-q_w, \qquad \text{for } y=0$$

$$u=Ue, \quad v=\omega=0, \quad T=T_\infty, \quad \text{for } y\to\infty \tag{1}$$

To solve the basic boundary-layer equations, we introduce pseudosimilarity variables (ξ,η), dimensionless rotating velocity function g, and dimensionless temperature θ (ISO), which are the same as those in Ref. 3. Besides, we also define a stream function $\psi(x,y)$ (for permeable surfaces) and a dimensionless temperature θ (CHF) as follows:

$$\theta=(T-T_\infty)Re_L^{1/2}/(q_w L/k) \tag{2}$$

$$\psi(x,y)=u_\infty L(2\xi/Re_L)^{1/2}f(\xi,\eta)-\int_0^x \frac{r}{L}v_w \mathrm{d}x \tag{3}$$

where the stream function ψ satisfies the continuity equation, where $ru=L\partial\psi/\partial y$, $rv=-L\partial\psi/\partial x$, and Re_L is the Reynolds number $Re_L=u_\infty L/\nu$.

By the dimensionless variable transformation mentioned above, the transformed governing equations and their boundary conditions are

$$f'''+ff''+\Lambda(1-f'^2)+\frac{2\xi}{r}\frac{\mathrm{d}r}{\mathrm{d}\xi}\left(\frac{r^2\Omega^2}{Ue^2}\right)g^2$$
$$-\alpha(\xi)f''=2\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \tag{4}$$

$$g''+fg'-gf'\left(\frac{4\xi}{r}\right)\frac{\mathrm{d}r}{\mathrm{d}\xi}-\alpha(\xi)g'$$
$$=2\xi\left(f'\frac{\partial g}{\partial\xi}-g'\frac{\partial f}{\partial\xi}\right) \tag{5}$$

$$Pr^{-1}\theta''+f\theta'-\alpha(\xi)\theta'$$
$$=2\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \tag{6}$$

with the boundary conditions

$$f=f'=0, \quad g=1, \quad \theta=1$$
$$\text{or } \theta'=-\sqrt{2\xi}/(rUe/Lu_\infty) \quad \text{for } \eta=0 \tag{7a}$$

$$f'=1, \ \theta=0, \ g=0 \ \text{for } \eta\to\infty \tag{7b}$$

where

$$\alpha(\xi)=\frac{v_w}{Ue}\frac{L}{r}\sqrt{2\xi}Re_L^{1/2}$$

and

$$\Lambda=\frac{2\xi}{Ue}\frac{\mathrm{d}Ue}{\mathrm{d}\xi}$$

Table 1 Local frictional coefficient $\frac{1}{2}C_f Re_R^{\frac{1}{2}}$ for a rotating sphere

		$B=1$			$B=4$			$B=10$		
Λ	x/R	Lee[a]	Hoskin[b]	Present method	Lee[a]	Hoskin[b]	Present method	Lee[a]	Hoskin[b]	Present method
0.48	0.474	1.2496	1.2497	1.2499	1.8170	1.8170	1.8182	2.8166	2.8165	2.8196
0.40	0.951	1.8403	1.8402	1.8400	2.6362	2.6359	2.6366	4.0444	4.0440	4.0462
0.30	1.215	1.7207	1.7203	1.7185	2.4023	2.4031	2.3990	3.6186	3.6218	3.6133
0.20	1.374	1.4780	1.4783	1.4732	1.9892	1.9953	1.9786	2.9144	2.9312	2.8930
0.10	1.486	1.2269	1.2336	1.2173	1.5644	1.5876	1.5373	2.1897	2.2451	2.1313

[a]Reference 3, 3 terms. [b]Reference 2, 4 terms.

Table 2 Values of $NuRe_R^{-\frac{1}{2}}$ for a rotating sphere

			$Pr=1$			$Pr=10$			$Pr=100$	
B	Λ	x/R	Lee[a]	Hoskin[b]	Present method	Lee[a]	Hoskin[b]	Present method	Lee[a]	Present method
1	0.5	—	0.9588	0.9589	0.9586	2.2363	2.2364	2.2359	4.9878	4.9947
	0.4	0.951	0.7998	0.7792	0.7993	1.8520	1.8571	1.8513	4.1151	4.1238
	0.3	1.215	0.6961	0.7064	0.6966	1.5994	1.6029	1.6004	3.5375	3.5543
	0.2	1.374	0.6171	0.6275	0.6195	1.4084	1.4173	1.4102	3.0981	3.1217
	0.1	1.486	0.5510	0.5557	0.5559	1.2551	1.2600	1.2511	2.7444	2.7586
10	0.5	—	1.1141	1.0914	1.1142	2.7718	2.7635	2.7701	6.3845	6.3848
	0.4	0.951	0.9218	0.9264	0.9215	2.2695	2.2722	2.2684	5.2019	5.2138
	0.3	1.215	0.7904	0.8042	0.7924	1.9234	1.9363	1.9278	4.3812	4.4205
	0.2	1.374	0.6825	0.7223	0.6902	1.6434	1.6640	1.6543	3.7114	3.7852
	0.1	1.486	0.5776	0.5905	0.5982	1.3912	1.4180	1.4044	3.1053	3.2048

[a]Reference 3, 3 terms. [b]Reference 2, 4 terms.

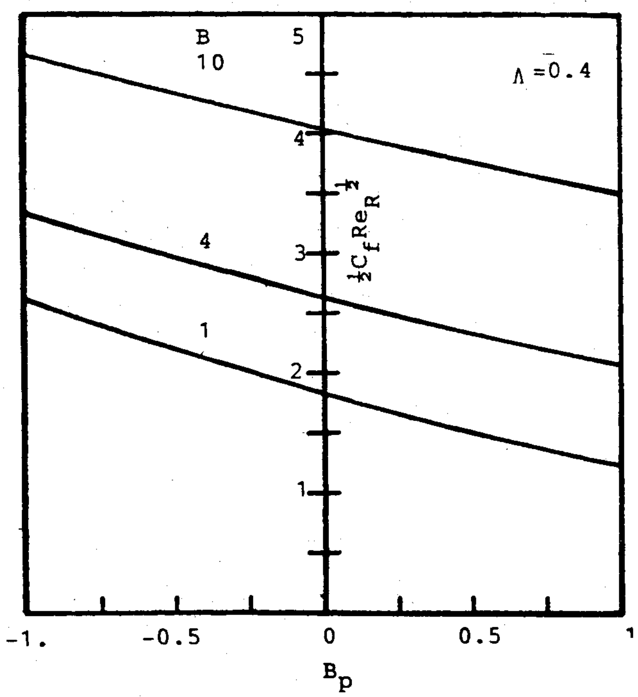

Fig. 1 Local friction factor as a function of the mass-transfer parameter B_p for various rotation parameters B.

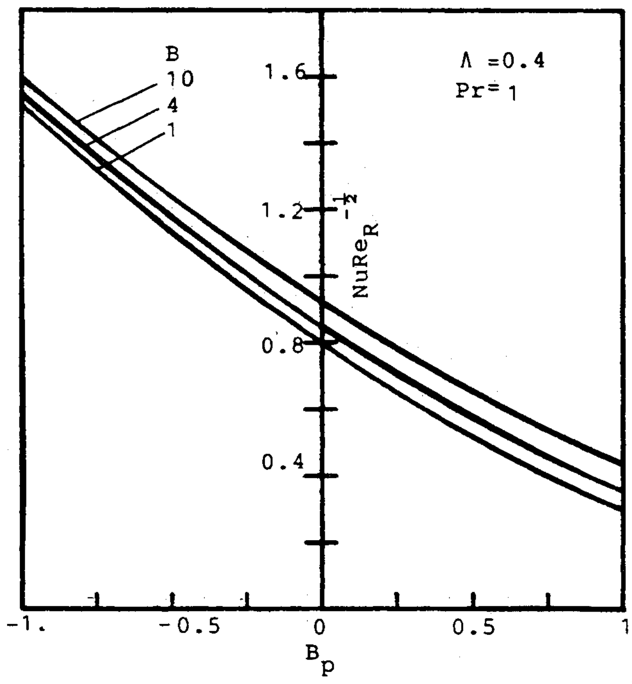

Fig. 2 Local Nusselt number as a function of the mass-transfer parameters B_p for various rotation parameters B of isothermal surface.

Application to a Rotating Sphere

For a spherical body, it is known from potential flow theory that $Ue/u_\infty = 3/2\ \sin x/R$ and, from the geometry, that $r/R = \sin x/R$. The quantities ξ, $(r^2\Omega^2/Ue^2)\ (2\xi/r)\mathrm{d}r/\mathrm{d}\xi$, $(4\xi/r)\mathrm{d}r/\mathrm{d}\xi$, and Λ are expressed in Ref. 3, and $\alpha(\xi)$ can be written as follows:

$$\alpha(\xi) = B_p\sqrt{2\xi}/(1.5\sin^2\beta) \tag{8}$$

where

$$B_p = v_w Re_L^{\frac{1}{2}}/u_\infty, \qquad L=R, \quad \text{and} \quad \beta = x/r$$

The physical quantities of interest are the local friction factor C_f and the local Nusselt number Nu, which are expressed as

$$\tfrac{1}{2}C_f Re_L^{\frac{1}{2}} = 2.25\sin^3\beta(2\xi)^{-\frac{1}{2}}f''(\xi,0) \tag{9}$$

$$NuRe_L^{-\frac{1}{2}} = 1.5\sin^2\beta(2\xi)^{-\frac{1}{2}}\theta'(\xi,0) \quad \text{(ISO)} \tag{10}$$

$$NuRe_L^{-\frac{1}{2}} = 1/\theta(\xi,0) \quad \text{(CHF)} \tag{11}$$

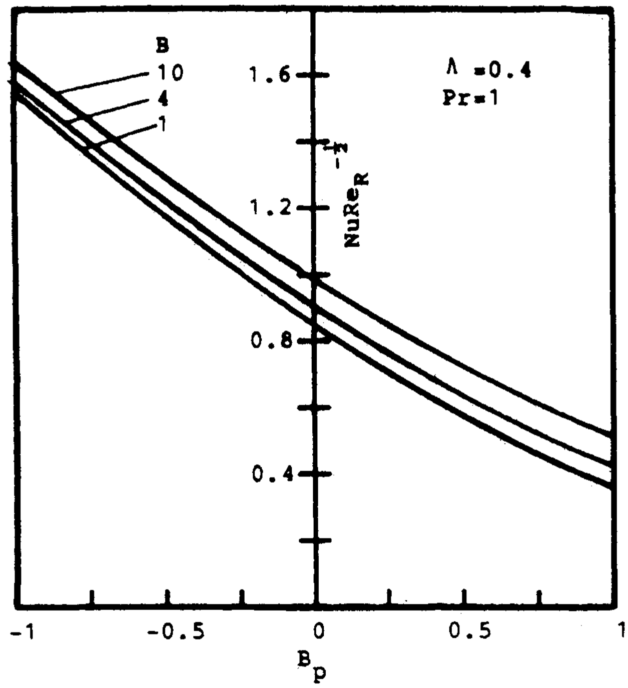

Fig. 3 Local Nusselt number as a function of the mass-transfer parameter B_p for various rotation parameters B of constant heat-flux surface.

Results and Discussion

We use a two-point finite difference method to solve the system (4-7). This is a very efficient numerical method developed by Cebeci and Bradshaw.[4]

Figure 1 shows that for a given value of rotation parameter B $(4/9(R\Omega/u_\infty)^2\Lambda)$, the wall shear decreases monotonically with B_p as the suction intensity decreases from -1 to 0 and the injection intensity increases from 0 to 1. The ordering of the curves with rotation parameter B is governed by the angular velocity of the rotating body. An increase in the value of the rotation parameter yields an increase in the value of the local friction coefficient when the other conditions are fixed.

Figures 2 and 3 show that for a given value of rotation parameter B, the local surface heat-transfer rate of isothermal surface or constant-heat-flux surface, like that of the wall shear, decreases monotonically as the suction intensity decreases and as the injection intensity increases. The ordering of the curve is the same as in Fig. 1. The higher rates of the surface heat transfer associated with larger values of rotation parameter B are due to the increase in the axial velocity that results in enhancing the convective heat-transfer rate between the body and the fluid.

For comparison of our numerical results for wall shear and Nusselt number without blowing and suction, Lee et al.'s[3] Merk-type series formula and Hoskin's[2] Blasius-type solutions were used. The results are included in Tables 1 and 2 and show excellent agreement with our result.

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Aluminum Combustion at 40 Atmospheres Using a Reflected Shock Wave

J.F. Driscoll,* J.A. Nicholls,† V. Patel,‡
B.K. Shahidi,‡ and T.C. Liu§
University of Michigan, Ann Arbor, Michigan

Introduction

IT has been found that an ideal way to study the reactions between aluminum and the gases N_2, H_2, Cl_2, and O_2 at elevated pressures up to 40 atm and temperatures up to 5150 K is to use a reflected shock wave within a shock tube. A technique was developed in which a heated, thin aluminum sheet is placed on the end wall of a single pulsed shock tube filled with the desired gaseous reactants. After a shock wave reflects off the end wall, elevated pressures and temperatures up to 40 atm and 5000 K were achieved. Pressure is constant during the run time of 10 ms which is more than adequate to study the fast reactions that occur. A diffusion flame is formed between the aluminum sheet and the hot reactant gases. Emission spectroscopy was used to identify some important intermediate species in the flame. The time histories of the species concentrations obtained in this way can yield useful chemical kinetics data.

In the past, extensive studies of aluminum-based rocket propellant fuels have conducted in order to develop ways to control the amount and size of Al_2O_3 particles in rocket exhausts. Aluminum particles can increase plume visibility, radiative heating of the nozzle, and two-phase flow losses. Controllable laboratory experiments that have been used to study aluminum combustion are the fast flow reactor,[1,2] the burning of aluminum ribbons,[3,4] and the use of incident shock waves in a shock tube.[5,6] However, the first two techniques have been limited to pressures of a few atmospheres or less. Reactions that occur at elevated pressures in rocket motors may differ from those that occur at 1 atm. The use of incident shock waves expose the aluminum to supersonic velocities that may not be be desired. The use of a reflected shock, as described in this Note, provides a stagnant gas environment at much higher pressures and temperatures than even incident shock waves can provide.

Experimental Method

An aluminum sample (2 cm × 2 cm) having thickness that varied from 0.01 to 0.1 cm was mounted on the end wall of a single-pulse shock tube.[7] The 4.7 m long driver section has an i.d. of 7.4 cm, and the 6.7 m long driven section has inner dimensions of 7.4 cm × 4.8 cm. The driven gas was either pure O_2 or the mixture $0.1\ N_2 + 0.4\ H_2 + 0.1\ Cl_2 + 0.4\ O_2$, which has the same elemental composition as ammonium perchlorate. For a driven gas pressure of 0.026 atm and a helium driver gas pressure of 102 atm, the incident shock Mach number was 10.5, and the temperature and pressure behind the reflected shock were 5150 K and 40 atm,

Presented as Paper 84-1201 at the AIAA/SAE/ASME 20th Joint Propulsion Conference, Cincinnati, OH, June 11-13, 1985; revision received Aug. 14, 1985.

*Associate Professor, Aerospace Engineering. Member AIAA.
†Professor, Aerospace Engineering. Fellow AIAA.
‡Research Assistant.
§Visiting Scientist, Institute of Aeronautics and Astronautics, Beijing, China.